Optical Component Characterization: A Linear Systems Approach

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1 Introduction

As modern optical communications systems are designed to fill more of the available bandwidth in optical fibers, performance qualification of passive optical components that support this expansion becomes more difficult and more costly. Recent advances in interferometric component measurement techniques have made it possible to completely characterize a component over a broad range of wavelengths (100 nm) in a very short time (200 ms/nm) by measuring the component's complete linear transfer function.[2] Figure 1 shows an idealized linear systems model of the transmission of an incident signal $E_{in}(t)$ through N concatenated passive components, each with transfer function $H_i(\omega)$.

The output signal, or field, is given by:

$$E_{out}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)E_{in}(\omega)e^{i\omega t}d\omega \tag{1}$$

where the system response (or transfer function) is given by a multiplication in the frequency domain, $H(\omega) = H_1(\omega)H_2(\omega)\cdots H_N(\omega)$, $H(\omega) = \int_{-\infty}^{\infty} dt\, h(t)e^{-i\omega t}$, and h(t) is the impulse response of the system. The transfer function of any given component has the form

$$H_i(\omega) = \rho_i(\omega)e^{i\phi_i(\omega)}. \tag{2}$$

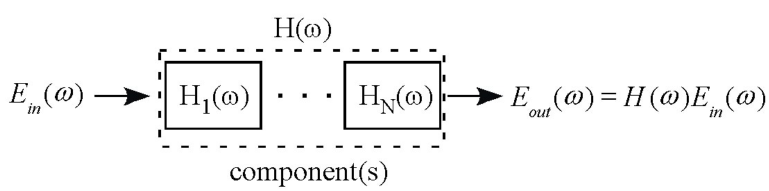

Figure 1: Linear systems representation of the transmission of a multiplexed signal through concatenated linear components. Each linear component has a transfer function, $\mathrm{H_i}(\omega)$, and the whole system can be represented by a single transfer function, $\mathrm{H}(\omega) = \mathrm{H_1}(\omega)\mathrm{H_2}(\omega)\cdots\mathrm{H_N}(\omega)$.

Once the complete transfer function (with amplitude *and* phase information) is known, the components performance in a fiber-optic system is completely determined.

Because the polarization state of an optical signal in a fiber can always be described as a linear combination of two orthogonal polarization states, a single-mode fiber must be described mathematically as a two-mode device. This is accomplished by using a 2x2 matrix as the fiber's transfer function:

$$\vec{E}_{out}(\omega) = \begin{pmatrix} a(\omega)e^{i\phi_a(\omega)} & b(\omega)e^{i\phi_b(\omega)} \\ c(\omega)e^{i\phi_c(\omega)} & d(\omega)e^{i\phi_d(\omega)} \end{pmatrix} \vec{E}_{in}(\omega). \tag{3}$$

This matrix was originally developed to describe the evolution of the polarization-state of the electric-field in free-space- and bulk-optical systems and is known as the Jones matrix[1, 3]. Note that the Jones matrix for any concatenation of components is formed by a multiplication of the individual Jones matrices. From the Jones matrix of an optical component, one can derive any linear parameter that characterizes the device [e.g. Insertion Loss (IL), Polarization Dependant Loss (PDL), Group Delay (GD), Chromatic Dispersion (CD), Polarization Mode Dispersion (PMD), 2nd order PMD, etc.].

Chromatic dispersion is often the linear quantity of greatest interest for fiber links because, as bit rates have increased, more links have become dispersion-limited as opposed to loss-limited. Fortunately, dispersion compensating fiber (or other forms of dispersion compensation) provides a means of correcting this. CD is derived by expanding the phase of the optical transfer function of the fiber into a Taylor series

$$\phi(\omega - \omega_o) \approx \phi_o + \tau_g(\omega - \omega_o) + {}^1\!/\!_2 CD(\omega - \omega_o)^2 + \cdots, \tag{4}$$

where

$$\begin{aligned} \phi_o &= \text{const.}, \\ \tau_g &\propto \text{group delay}, \\ CD &\propto \frac{\partial}{\partial \omega}(\text{group delay}). \end{aligned} \tag{5}$$

Generally, the CD of a fiber link is not measured directly. Instead the group delay is measured as a function of frequency, and the chromatic dispersion is calculated as a numerical derivative of the group delay. The spectral features of a link are typically very broad, having dominant features that vary on the order of tens of nanometers. As a result, over any particular channel the CD will be constant and the above Taylor expansion to second order is valid (see Fig. 2). In short, the phase response of optical fiber *links* can be modelled and well managed by using CD as the descriptor for any given component in a broad band link.

Traditionally, optical components have been characterized by loss measurements, in particular, insertion loss (average loss over all polarization states) and Polarization Dependent Loss (Maximum change in loss over all incident polarization states). Optical filters in Dense Wavelength Division

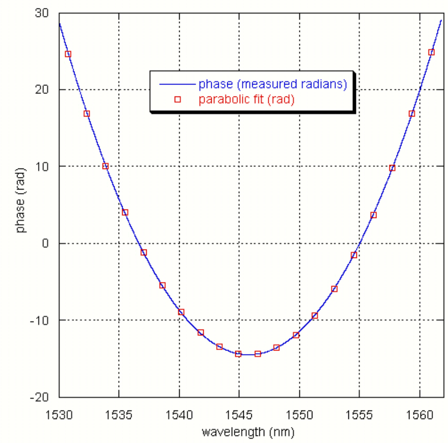

Figure 2: An example of the phase response of an optical fiber link over approximately 1.5 THz. Note that that parabolic nature of the phase lends itself to a second order Taylor expansion.

Multiplexing (DWDM) systems are generally characterized by these parameters as a function of the incident wavelength (see Fig. 3). From this spectral response, quantities such as channel loss, channel ripple, and channel-to-channel isolation can be calculated. As the density of information carried in a single fiber increases, this simple loss characterization becomes insufficient to predict the final performance of a component in an optical communications link.

The success of chromatic dispersion and group delay in the characterization of fiber links has led to the adoption of these same parameters as characteristics of DWDM filters and components. Unfortunately, the approximation of the chromatic dispersion as a constant over a channel does NOT hold for the vast majority of DWDM components. Figure 3 illustrates this point by displaying the amplitude and phase of the response of a typical filter.

Most DWDM filters exhibit relatively complicated phase structures. As a result, it becomes very difficult to tie specific values of group delay or chromatic dispersion to important parameters such as noise penalty or bit-error-rate. Fortunately, electromagnetic filters have a long history and a large body of work addressing them. Here, we would like to apply some very simple linear systems concepts to optical systems, and in particular, to the effects of non-ideal filters. The following section addresses amplitude and phase errors using the scalar model of signal propagation in a fiber, and in Section 3, polarization errors due to birefringence using the full vector model of the electric field are treated.

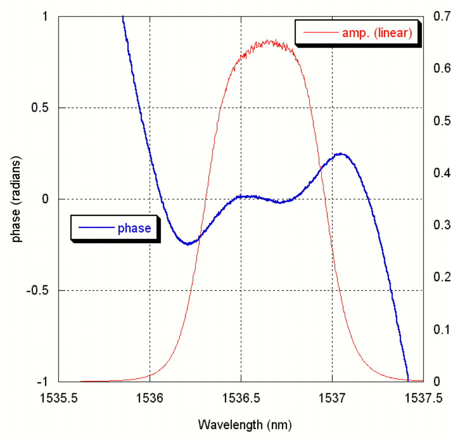

Figure 3: An example of the phase and amplitude response of a typical DWDM filter over approximately 130 GHz. Note that the phase is not well-represented by a second order Taylor series in this case.

2 Scalar model and the equivalence of phase and amplitude errors.

If we hypothesize an ideal filter, and wish to send an optical bit-stream through the filter, we can find the output bit stream quite simply using Eq. 1. Note that the format of the bit sequence (NRZ, RZ, etc.) need not be specified for this analysis. We can then introduce a non-ideal filter that we assume is close to the ideal one, so that we can take the errors to be small. This non-ideal filter will then have some amplitude perturbation, and some phase perturbation:

$$G(\omega) = H(\omega)[1 + \Delta\rho]e^{i\Delta\phi} \approx H(\omega)[1 + \Delta\rho][1 + i\Delta\phi], \tag{6}$$

where $G(\omega)$ is the actual filter response, $H(\omega)$ is the response of an ideal filter, $\Delta\rho$ is the amplitude error, and $\Delta\phi$ is the phase error to all orders. Using Eq. 6 in Eq. 1 and carrying out some simplifications, we see that there is an equivalence between the amplitude error and the phase error in the small error approximation:

$$E_{out}(t) \approx E_{ideal}(t) + \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)E_{in}(\omega)[\Delta\rho + i\Delta\phi]e^{i\omega t}d\omega. \tag{7}$$

Given this expression for the signal strength, we can define a quantity proportional to the optical power at the receiver, $P \equiv |E_{out}(t)|^2$. From Eq. 7, we get

$$P = |E_{ideal}(t)|^2 + \mathrm{Re}\left\{\frac{E^*_{ideal}(t)}{\pi}\int_{-\infty}^{\infty} H(\omega)E(\omega)[\Delta\rho + i\Delta\phi]e^{i\omega t}d\omega\right\} + O^2, \tag{8}$$

where the terms that are of order 2 and higher in the error have been neglected.

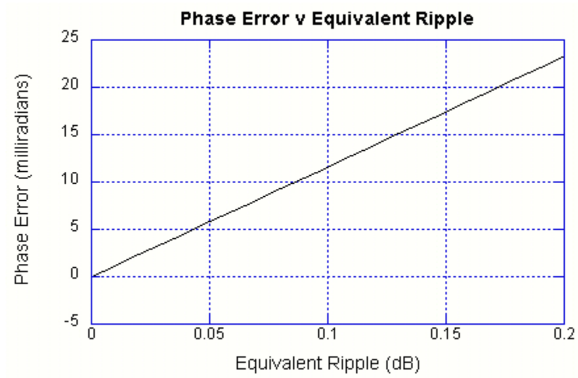

Figure 4: Phase error that is equivalent to a given amount of insertion-loss ripple in the pass-band. This figure can be interpreted as follows: for a given ripple specification (horizontal axis) over the pass-band of a component, there is a phase error (vertical axis) that must not be exceeded if the component is to function properly in a system.

Now, a simple offset (constant error) in the phase value will have no effect on the power of the signal versus time, and a linear error in the phase amounts to a time shifting of the signal, which also has no effect on the bit error rate. These terms can be grouped into $H(\omega)$ and, therefore, do not represent phase error. The quantity $\Delta\phi$ contains any remaining error in the phase and thus degrades the optical signal. Note that the derivative of the phase does not appear in the integral, only the total phase error. The phase parameter $\Delta\phi$ is commonly referred to as the deviation from linear phase. This parameter has units of radians and is specified only over the pass-band of a given device.

Since amplitude ripple is given by $20\log(1+\Delta\rho)$, we can use the equivalence relationship found above to construct a plot of the phase error that is equivalent to some insertion loss ripple specification in the pass band using:

$$\text{Equivalent Ripple} = 20\log(1+\Delta\phi). \tag{9}$$

This is shown in Fig. 4. Figure 4 helps to prevent over-specification of parameters. Here is an example: if one specifies the insertion loss ripple to be less than .05 dB, then the deviation from linear phase should be specified to be around 6 milliradians to ensure proper performance. If the phase error is allowed to be much larger, then meeting the tight ripple specification will NOT contribute to the performance of the communications link, and will only serve to reduce yields and drive up cost.

If we assume that the phase and amplitude errors are uncorrelated (not altogether a safe assumption, but convenient for the moment), we can approximate the total deleterious effect from the errors to be the vector sum of the ripple (in percent) and the phase error (in radians): error $= \sqrt{|\Delta\rho|^2 + |\Delta\phi|^2}$. An estimate of the noise penalty can then be found to be

$$\text{noise penalty} = 20\log[1/(1-\text{error})]. \tag{10}$$

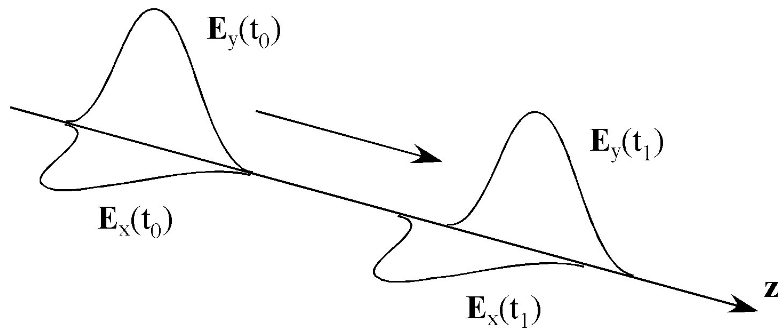

Figure 5: An illustration of the DGD interpretation of PMD.

Finally, we should note that the total error is the product of the signal spectrum and the error spectrum. Therefore, the effects of a particular filter on a Bit Error Rate (BER) measurement may be dependent on the bit sequence or the center wavelength of the laser relative to the filter center wavelength.

3 Polarization Effects

As the bit rate per channel increases beyond 10 GB/s, signal spectra become wide enough that polarization effects begin to affect signal quality. The quantity used for quantifying these polarization effects has generally been referred to as Polarization Mode Dispersion (PMD), or Differential Group Delay (DGD), as it can be interpreted as different polarization components experiencing different delays as they propagate. This interpretation can be depicted in the time domain as shown in Fig. 5. If the PMD remains constant over the signal bandwidth, then this interpretation works well. If the PMD varies over the channel, however, and particularly if the principal states vary or the sign of the PMD changes, then the differential delay model becomes difficult to apply. In this case it is more useful to note that PMD is actually a relative phase shift between signal components propagating in the principal polarization states of the device. (The delay is given by the derivative of this phase shift with respect to frequency.) This phase shift means that at a given frequency, the vector sum of the light contained in the principal states will change upon propagation, thus the overall polarization state of the light at that frequency will be changed. At the receiver, all frequency components of the signal must be in the same polarization state, otherwise they will not interfere to reproduce the signal pulse. As illustrated in Fig. 6, different relative phase shifts at different optical frequencies mean that the two ends of the pulse spectrum will not match in polarization. This polarization error leads to a degradation of the signal pulse.

In order to understand polarization effects in the context of linear systems theory, we will now use the full vector description of light and the Jones matrix description of the component. In this case we will assume no amplitude or phase errors in the device matrix, and introduce only a polarization

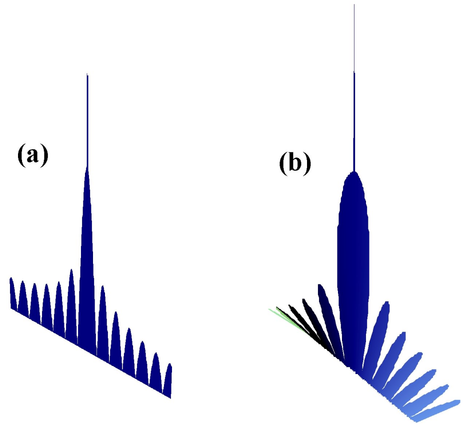

Figure 6: (a) A signal spectrum upon propagation through a system with no PMD. (b) When PMD is present, different frequency components of the signal change polarization state relative to one another due to varying phase shifts experienced by the principal polarization states. This is illustrated by a "twisting" of the side bands away from the polarization state defined by the center of the signal spectrum.

error due to birefringence. In order to keep the math simple, we choose to represent the input electric field in terms of the basis defined by the two principal states of the device. This allows us to represent the ideal transfer matrix $\overline{\overline{H}}(\omega)$ as

$$\overline{\overline{H}}(\omega) = H(\omega)\overline{\overline{I}} \tag{11}$$

where $H(\omega)$ is a scalar function of frequency and $\overline{\overline{I}}$ is the identity matrix. The departure from the ideal transfer matrix that includes a small amount of birefringence is given by

$$\overline{\overline{G}}(\omega) = \overline{\overline{H}}(\omega) \begin{bmatrix} e^{i\Delta\beta/2} & 0 \\ 0 & e^{-i\Delta\beta/2} \end{bmatrix}. \tag{12}$$

Furthermore, we will assume a worst case input field where there is equal power in each of the principal states. In this case, we may express the input electric field $\vec{E}_{\text{in}}(\omega)$ as

$$\vec{E}_{\text{in}}(\omega) = E_{\text{in}}(\omega) \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{13}$$

Upon propagation, the output field becomes

$$\vec{E}_{\text{out}}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{\overline{G}}(\omega)\vec{E}_{\text{in}}(\omega)e^{i\omega t}d\omega. \tag{14}$$

Assuming that the birefringence is small, we can expand the exponentials in a Taylor series. We will keep terms through second order, and denote this approximation to $\vec{E}_{\text{out}}(t)$ as $\vec{E}_2(t)$:

$$\vec{E}_2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{\overline{H}}(\omega) \begin{bmatrix} 1 + i\Delta\beta/2 - \Delta\beta^2/4 & 0 \\ 0 & 1 - i\Delta\beta/2 - \Delta\beta^2/4 \end{bmatrix} \vec{E}_{\text{in}}(\omega)e^{i\omega t}d\omega. \tag{15}$$

This can be recast as

$$\vec{E}_2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{\overline{H}}(\omega)\vec{E}_{\text{in}}(\omega)e^{i\omega t}d\omega + \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{\overline{H}}(\omega) \begin{bmatrix} i\Delta\beta/2 - \Delta\beta^2/4 & 0 \\ 0 & -i\Delta\beta/2 - \Delta\beta^2/4 \end{bmatrix} \vec{E}_{\text{in}}(\omega)e^{i\omega t}d\omega, \tag{16}$$

where it is apparent that the first term is the response to the ideal filter, and the second term represents the error due to birefringence. We will now make use of the simplifications introduced in Eqs. 11 and 13, which represent the worst case scenario for signal degradation due to birefringence:

$$\vec{E}_2(t) = \vec{E}_0(t) + \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\begin{bmatrix} i\Delta\beta/2 - \Delta\beta^2/4 \\ -i\Delta\beta/2 - \Delta\beta^2/4 \end{bmatrix} E_{\text{in}}(\omega)e^{i\omega t}d\omega, \tag{17}$$

where $\vec{E}_0(t)$ is the output signal of the ideal filter. Equation 17 is analogous to Eq. 7. Now in order to determine the effects of PMD , it is necessary to calculate the detected power. This is given by

$$\begin{aligned} P_2(t) &= \left|\vec{E}_0(t)\right|^2 + \frac{1}{\pi}\text{Re}\left\{\vec{E}_0(t)\cdot\int_{-\infty}^{\infty} H^*(\omega)\begin{bmatrix} -i\Delta\beta/2 - \Delta\beta^2/4 \\ i\Delta\beta/2 - \Delta\beta^2/4 \end{bmatrix} E^*_{\text{in}}(\omega)e^{-i\omega t}d\omega\right\} \\ &+\left|\frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\begin{bmatrix} i\Delta\beta/2 - \Delta\beta^2/4 \\ -i\Delta\beta/2 - \Delta\beta^2/4 \end{bmatrix} E_{\text{in}}(\omega)e^{i\omega t}d\omega\right|^2. \end{aligned} \tag{18}$$

After some manipulation, it can be shown that the last term in $P_2(t)$ has only error terms of order 4 and greater and can thus be neglected. Note that the assumptions made in Eqs. 11 and 13 imply that

$$\vec{E}_0(t) = E_0(t)\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \tag{19}$$

so $P_2(t)$ can be expressed as

$$\begin{aligned} P_2(t) &= \left|\vec{E}_0(t)\right|^2 + \frac{1}{\pi}\text{Re}\left\{E^*_0(t)\int_{-\infty}^{\infty}\left(i\frac{\Delta\beta}{2} - \frac{\Delta\beta^2}{4} - i\frac{\Delta\beta}{2} - \frac{\Delta\beta^2}{4}\right)H(\omega)E_{\text{in}}(\omega)e^{i\omega t}d\omega\right\} \\ &= \left|\vec{E}_0(t)\right|^2 - \frac{1}{2\pi}\text{Re}\left\{E^*_0(t)\int_{-\infty}^{\infty}\left(\Delta\beta^2\right)H(\omega)E_{\text{in}}(\omega)e^{i\omega t}d\omega\right\}. \end{aligned} \tag{20}$$

Here we see that the polarization error is a second order effect, as the first order terms in $\Delta\beta$ cancel. Equation 20 shows how polarization error due to birefringence in an optical component directly affects the measured bit pulse at the receiver. This is analogous to the effects of both amplitude and phase error, as discussed in the previous section. It is the square of the polarization error, however, which affects signal quality, whereas amplitude and phase error enter the equations linearly. Therefore, less stringent requirements need to be placed on the polarization error of the system than the phase error. To expand upon the example of the previous section, an amplitude ripple requirement of < 0.05 dB requires a phase error of < 6 milliradians, and a polarization error of < 77 milliradians.

Having measured the complete Jones matrix of an optical component, it is possible to extract the polarization error and plot it as shown in Fig. 7. This plot relates directly to Fig. 6(b), as it displays the amount of “twisting” of the polarization state in radians incurred by the signal spectrum. To qualify an optical component using this type of data, one need only check whether or not the polarization error reaches a threshold value over the passband of interest.

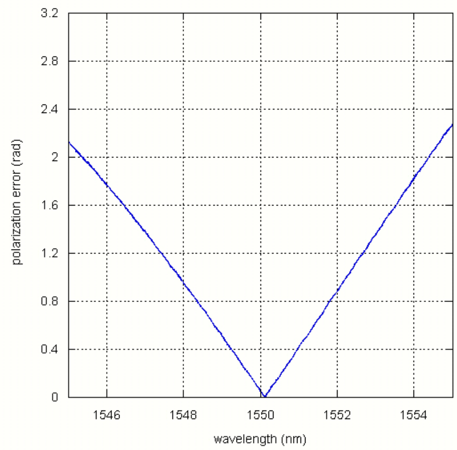

Figure 7: The measured polarization error of a standard broadband PMD source.

4 Conclusions

We have introduced a simple, linear systems formulation of the effect of passive components on a communications signal in an optical fiber. Central to the development is the fact that components can be qualified quickly and precisely by interferometric measurement of the amplitude *and* phase of the transfer function of the device, and that both amplitude (PDL) and phase (PMD) related polarization effects can be can be gathered by measurement of the Jones matrix of the device. Once the Jones matrix is known, the component's effect on an optical signal is completely determined.

Currently optical devices are specified using four major quantities: insertion loss, group delay (or chromatic dispersion), PDL, and PMD. This set of quantities may not, however, provide a direct indication of device performance in a communication system. Full knowledge of the Jones matrix of a component allows for rigorous component qualification based on more fundamental device parameters: deviation from linear phase and polarization error. These parameters, along with insertion loss and PDL, are straightforward and accurate indicators of device performance. Furthermore, this method of component qualification avoids the problems of over-specification that arises when using group delay and PMD. This, combined with the speed of interferometric measurement, will help to reduce the cost associated with component specification and qualification.

References

[1] E. Hecht. *Optics*. Addison-Wesley, Reading, Massachusetts, second edition, 1990.

[2] For more information, please visit http://www.lunatechnologies.com/.

[3] For more information on the Jones matrix and its uses in characterizing optical components, please visit http://www.lunatechnologies.com/files/23jonesintro.pdf.