

Second-Order PMD in Optical Components

Brian J. Soller
sollerb@lunatechnologies.com

Luna Technologies, 3157 State Street, Blacksburg, VA 24060

May 13, 2005

1 Introduction

Second-order PMD (PMD²) is defined as the frequency derivative of the PMD vector, $\mathbf{\Omega}(\omega) = \Delta\tau(\omega)\hat{q}(\omega)$, [1] where $\Delta\tau(\omega)$ is the differential group delay (DGD = first-order PMD) between the two principle states of polarization (PSP's) and \hat{q} is a unit vector that points in the direction of the fast principle axis. Hence, the second-order PMD is given by

$$\text{PMD}^2 = \frac{d\mathbf{\Omega}(\omega)}{d\omega} = \frac{d\Delta\tau(\omega)}{d\omega}\hat{q}(\omega) + \Delta\tau(\omega)\frac{d\hat{q}(\omega)}{d\omega}. \quad (1)$$

The two terms in the above equation are often treated separately by breaking the second-order PMD into parallel and perpendicular components:

$$\mathbf{\Omega}_{\omega\parallel} = \frac{d\Delta\tau}{d\omega}\hat{q} \quad (2)$$

$$\mathbf{\Omega}_{\omega\perp} = \Delta\tau\frac{d\hat{q}}{d\omega}, \quad (3)$$

where $\mathbf{\Omega}_{\omega\parallel}$ causes polarization dependent chromatic dispersion and $\mathbf{\Omega}_{\omega\perp}$ represents a frequency dependent rotation of the PSPs of the DUT, both of which, in turn, causes signal depolarization. Also, we've used the short-hand $\mathbf{\Omega}_{\omega} = \frac{d\mathbf{\Omega}}{d\omega}$.

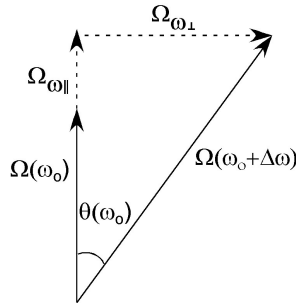


Figure 1: Vector picture of the PMD vector $\mathbf{\Omega}(\omega)$, and the components of its derivative, $\mathbf{\Omega}_{\omega\parallel}$ and $\mathbf{\Omega}_{\omega\perp}$.

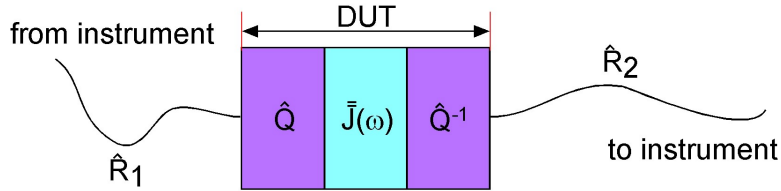


Figure 2: Transfer matrices that comprise a DUT with Jones matrix \bar{J} .

2 The Jones Matrix Method

Knowledge of a component's Jones matrix allows for straightforward calculation of that component's second-order PMD.[2] For the configuration in Fig. 2, the measured Jones matrix, $\bar{M}(\omega)$, is a two-by-two matrix with four complex entries and can be written

$$\bar{M}(\omega) = \hat{R}_2 \hat{Q}^{-1}(\omega) \bar{J}(\omega) \hat{Q}(\omega) \hat{R}_1, \quad (4)$$

where \hat{R}_1 and \hat{R}_2 represent non-frequency-dependent unitary rotation matrices corresponding to the action of the fiber leads on the polarization state of the electric field in the fiber, $\hat{Q}(\omega)$ is a unitary matrix that describes the frequency-dependent rotation of the input electric field vector into (and out of) the basis of eigenstates of the DUT, and $\bar{J}(\omega)$ is the frequency dependent Jones matrix of the DUT in the basis of its eigenstates (that is, the basis in which it is diagonal). Note that the eigenstates of $\bar{J}(\omega)$ are *not* the PSP's of the DUT. Rather, the PSP's are the eigenvectors of the product $\bar{M}_\omega \bar{M}^{-1}$. [3]

For most component measurement systems, the fiber leads will be relatively short (< 30 m) and will have very small values of residual PMD that does not vary substantially with frequency. Under such conditions, the approximation that $\hat{R} \neq \hat{R}(\omega)$ will be valid.

2.1 Differential Group Delay

Given $\bar{M}(\omega)$, the DGD is calculated by finding the eigenvectors of $\bar{M}_\omega \bar{M}^{-1}$. [3] By using $\bar{M}_\omega \bar{M}^{-1}$ instead of $\bar{M}_\omega \bar{M}^\dagger$, we allow for PDL in \bar{J} . The imaginary parts of the eigenvalues of the product $\bar{M}_\omega \bar{M}^{-1}$ are the group delays associated with the two PSP's. If ρ_1 and ρ_2 denote the two eigenvalues, then the group delay through the DUT is the average

$$\tau_g = 1/2 \text{Im}[\rho_1 + \rho_2], \quad (5)$$

and the DGD is given by

$$\Delta\tau = \text{Im}[\rho_1 - \rho_2]. \quad (6)$$

An example of the DGD of a 50:50 fiber-optic coupler is shown in Fig. 3. The Jones matrix used to produce all of the data in this paper was measured using an Optical Vector Analyzer.

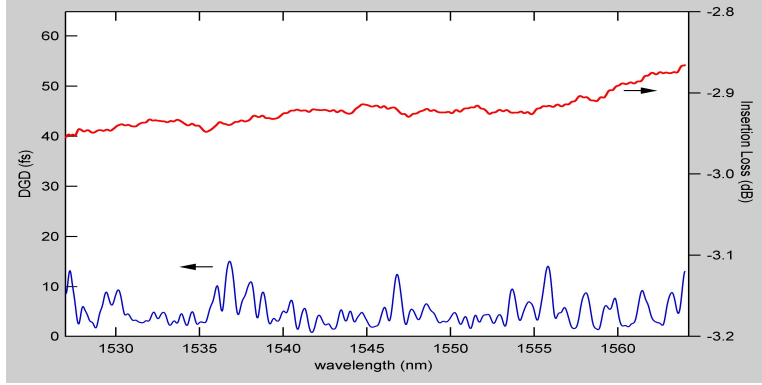


Figure 3: DGD and IL of 50:50 coupler

2.2 Principle States of Polarization

If $|\mathbf{s}_1\rangle$ and $|\mathbf{s}_2\rangle$ are the eigenvectors of $\bar{\bar{M}}_\omega \bar{\bar{M}}^{-1}$, then they are also the PSP's of the DUT. The unit vector \hat{q} is found by identifying the eigenstate associated with the fast principle axis and casting that state in a Stokes space. If

$$|\mathbf{s}_1\rangle = (A_x e^{i\delta_x}, A_y e^{i\delta_y}), \quad (7)$$

then the Stokes vector corresponding that PSP is[4]

$$\mathbf{S} = \begin{pmatrix} A_x^2 - A_y^2 \\ A_x A_y \sin(\delta) \\ A_x A_y \cos(\delta) \end{pmatrix}. \quad (8)$$

We can use Eq. (8) to find \hat{q}

$$\hat{q} = \frac{\mathbf{S}}{|\mathbf{S}|}. \quad (9)$$

The vector $\hat{q}(\omega)$ describes a locus of points on the Poincare sphere. The angular rate of change of \hat{q} ($= |\dot{\hat{q}}_\omega|$) describes the rotation rate of the PSP's as a function of frequency. A graphical depiction is shown in Fig. 4

The angle, $\theta(\omega)$, that describes the rotation of the PSP's is easily found using

$$\theta_n = \cos^{-1}(\hat{q}_{n-1} \cdot \hat{q}_n). \quad (10)$$

Figure 4 shows the PSP orientation angle, θ , calculated using Eq. (10) for a 50:50 coupler. The frequency derivative is shown on the same graph.

There is a built-in arbitrariness associated with the direction of the unit vector, \hat{q} , that is a result of the rotation matrices \hat{R}_1 and \hat{R}_2 being included in the measured transfer matrix, $\bar{\bar{M}}(\omega)$. This is best illustrated by example. consider a DUT with linear PSP's. Linear states are described in Stokes space as locations on the equator of the Poincare sphere. However, if a measurement of the DUT's Jones matrix includes the matrices \hat{R}_1 and \hat{R}_2 , the eigenvector of $|\mathbf{s}_1\rangle$ and $|\mathbf{s}_2\rangle$ of $\bar{\bar{M}}_\omega \bar{\bar{M}}^{-1}$

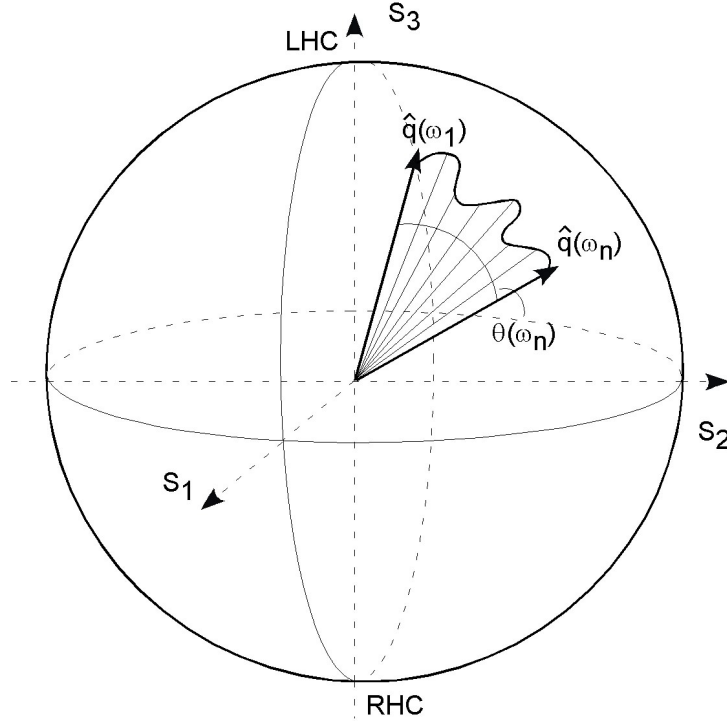


Figure 4: The unit vector, \hat{q} , represented as a locus of points on the Poincare Sphere. The angle θ describes the frequency dependent rotation of the PSP's

will not describe linear polarization. The linear PSP's of the DUT are in effect “mapped” to other polarization states, or locations on the Poincare sphere, by the arbitrary rotations of the attached fiber leads. Therefore, the absolute direction in which the vector \hat{q} points does not reflect the PSP's associated with $\bar{J}(\omega)$. However, since \hat{R}_1 and \hat{R}_2 are both unitary and frequency independent, the change in \hat{q} as a function of frequency is unaffected by \hat{R}_1 and \hat{R}_2 , and the second-order PMD calculated using \hat{q} will reflect that of the DUT.

3 Second-order PMD

For purposes of plotting, the two components of the second-order PMD are given by their absolute values

$$|\Omega_{\omega\parallel}| = \frac{d\Delta\tau}{d\omega} \quad (11)$$

$$|\Omega_{\omega\perp}| = \Delta\tau \frac{d\theta}{d\omega}. \quad (12)$$

Results of the second-order PMD of a 50:50 coupler for both parallel and perpendicular components calculated according to Eqs. (11) and (12) are shown in Fig. 5.

For the 50:50 coupler, neither contribution to the second-order PMD is large due to (a) the small intrinsic DGD value and (b) the well defined birefringence axis of the DUT.

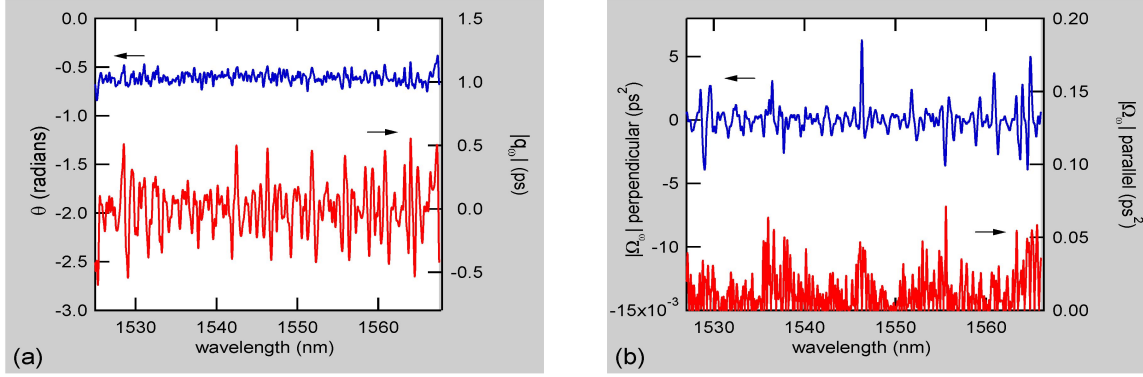


Figure 5: (a) DUT's orientation of PSP and calculated angular rate of change, $|\dot{q}_\omega| = \frac{d\theta}{d\omega}$, and (b) Calculated parallel and perpendicular components of the second-order PMD of the DUT using the Jones matrix eigenanalysis method.

An example of the two components of the second-order PMD of a thin film filter (TFF) is shown in Fig. 6 The return loss profile is overlaid in the graph for referencing purposes. The DGD is also shown. Notice that in the pass band, both components contribute similarly to the total second-order PMD.

4 Conclusions

This paper has outlined a simple method by which one can use measured Jones matrix data and extract second-order PMD information. It should be noted while results for first- and second-order PMD have been shown, this method is generally applicable to all orders of PMD.

The method has the advantage of being insensitive to the unitary properties of fiber leads connected to the DUT during Jones matrix measurement. The accuracy of the technique is directly related to the accuracy with which the Jones matrix is measured and the amount of residual birefringence in the fiber leads.

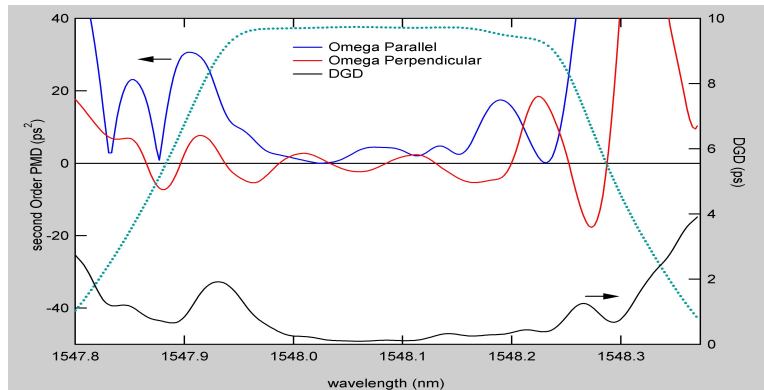


Figure 6: Calculated parallel and perpendicular components of the second-order PMD of the DUT using the Jones matrix eigenanalysis method.

A drawback to this method is that while the rotation rate of the PSP's is determined, the orientation of the PSP's is undetermined.

References

- [1] J. P. Gordon and H. Kogelnick, "PMD fundamentals: Polarization mode dispersion in optical fibers," *Proc. Nat. Acad. Sci.* **97**, 4541–4550 (2000).
- [2] H. Kogelnick, L. E. Nelson, J. P. Gordon, and R. M. Jopson, "Jones matrix for second-order polarization mode dispersion," *Opt. Lett.* **25**, 19–21 (2000).
- [3] B. L. Hefner, "Automated measurement of polarization mode dispersion using Jones matrix eigenlysis," *IEEE Photon. Technol. Lett.* **4**, 1066–1069 (1992).
- [4] M. Born and E. Wolf, *Principles of Optics* (Cambridge University Press, Cambridge, Massachusetts, 1999), seventh edn.